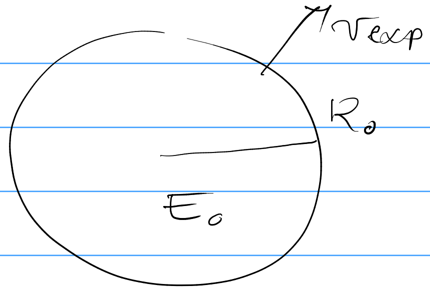
$E_0 \approx aT^4 V$ (gas internal energy is small)

$\ll E_{kin}$ (acceleration has happened)



$R_0 \approx$ few $R_{prog.}$

Just expanding $\dot{E} + P\dot{V} = 0$ adiabatic

with cooling & heating $= L_{heat} - L_{cool}$

$= \epsilon M - L$

Pressure $P \simeq P_{rad} = \frac{1}{3} aT^4 = \frac{1}{3}\frac{E_{rad}}{V}$

$$\dot{E} + \frac{1}{3}E\frac{\dot{V}}{V} = \epsilon M - L$$

$\div$

$$\frac{\dot{E}}{E} + \frac{1}{3}\frac{\dot{V}}{V} = \frac{\epsilon M}{E} - \frac{L}{E}$$

($\frac{\epsilon M}{E} \to 1/t_{heating}$, $\frac{L}{E} \to 1/t_{diffusion}$)

$E = aT^4 V$

$V = \frac{4\pi}{3}R^3$

$$4\frac{\dot{T}}{T} + \frac{\dot{V}}{V} + \frac{1}{3}\frac{\dot{V}}{V} = \frac{4\dot{T}}{T} + \frac{4\dot{R}}{R} = \frac{1}{t_{heat}} - \frac{1}{t_{diff}}$$

$$\frac{d\ln(TR)^4}{dt} = \frac{1}{t_{heat}} - \frac{1}{t_{diff}}$$

no heating / cooling $(TR)^4 = (TR)_0^4$

$$\Longrightarrow \frac{T}{T_0} = \frac{R_0}{R}$$ like the universe

$$E = aT^4 V \propto T^4 R^3 \propto 1/R$$

$$\Longrightarrow E = E_0 \frac{R_0}{R} = E_0 \frac{R_0}{R_0 + vt}$$

$$= E_0 \frac{1}{1 + t/t_{exp,0}} \qquad t_{exp,0} \equiv \frac{R_0}{v}$$

Add diffusion (cooling)

$$\tau_{diff} = \frac{R}{c}\frac{R}{l_{mfp}} = \frac{R^2 k\rho}{c} = \frac{kM}{\frac{4\pi}{3}Rc} = \frac{kM}{\beta Rc}$$

β = 13.8 for constant ρ

$$= \tau_{diff,0}\frac{R_0}{R} = \tau_{diff,0}\frac{1}{1+t/t_{exp,0}}$$

$$\frac{d\ln(TR)^4}{dt} = -\frac{1}{\tau_{diff}} = -\frac{1+t/t_{exp,0}}{\tau_{diff,0}}$$

$$\ln(TR)^4 = -\underbrace{\frac{t+\frac{1}{2}t^2/t_{exp,0}}{\tau_{diff,0}}} + \ln(TR)_0^4$$

$$\frac{T}{T_0} = \frac{R_0}{R}\left(e^{-\cdots}\right)^{1/4}$$

$$\Rightarrow E = E_0\frac{R_0}{R}e^{-(t\tau_{exp,0}+\frac{1}{2}t^2)/(\tau_{exp,0}\tau_{diff,0})}$$

$t < \tau_{exp,0} \Rightarrow$ exp. decay

$t > t_{exp,0} \Rightarrow$ gaussian

$$= E_0\frac{R_0}{R}\phi(t)$$

$\phi(t)$ → heating & cooling; $\frac{R_0}{R}$ → adiabatic exp

luminosity $L = \frac{E}{\tau_{diff}} = \frac{E_0\frac{R_0}{R}\phi(t)}{\tau_{diff,0}\frac{R_0}{R}} = L_0\phi(t)$

~ Gaussian ⇒ parabola in magnitudes

timescale $\sqrt{\tau_{exp,0}\tau_{diff,0}} = \sqrt{\frac{R_0}{v}\frac{kM}{\beta cR_0}} = \sqrt{\frac{kM}{\beta cv}}$

$$E_0 = \frac{1}{2}E_{SN}$$

$$L_0 = \frac{1}{2}\frac{E_{SN}}{M}R_0\frac{\beta c}{k} = \frac{1}{2}\frac{10^{51}}{2\times10^{33}}10^{14}\frac{13.8\ 3\times10^{10}}{0.4} \approx 10^{10}L_\odot$$

$\frac{E_{SN}}{M}$ → energy per unit mass $\propto T$, $\propto v^2$

R_0 → small ⇒ faint

$\frac{\beta c}{k}$ → transparency

Include heating

eg radioactive decay $\propto e^{-t/t_{decay}}$

$\rightarrow$ no longer analytic

$$\frac{d\ln (TR)^4}{dt} = \frac{1}{\tau_{heat}(t)} - \frac{1}{\tau_{cool}(t)} \equiv \frac{\dot{\phi}}{\phi}$$

$$(TR)^4 = (TR)_0^4 \, \phi(t)$$

$$\ldots \quad L = L_0 \, \phi(t)$$

lum has a maximum when

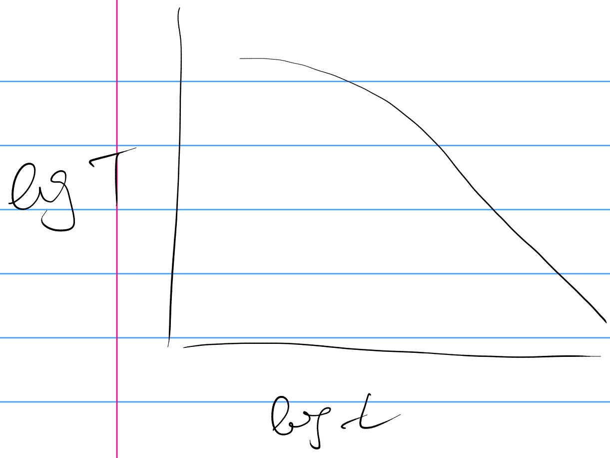
$$\frac{\dot{\phi}}{\phi} = \frac{1}{\tau_{heat}} - \frac{1}{\tau_{cool}} = 0$$

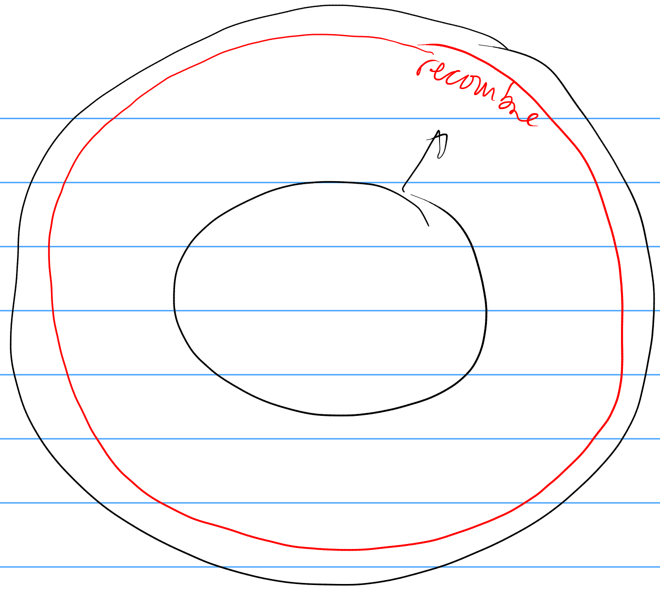
$$\times E: \quad \frac{E}{\tau_{heat}} - \frac{E}{\tau_{diff}} = 0 \Rightarrow L_{max} = \epsilon M$$

($\frac{E}{\tau_{heat}}$ = ϵM; $\frac{E}{\tau_{diff}} = L$, $\tau_{diff} = \tau_{cool}$)

$\Downarrow$

measure ^{56}Ni mass



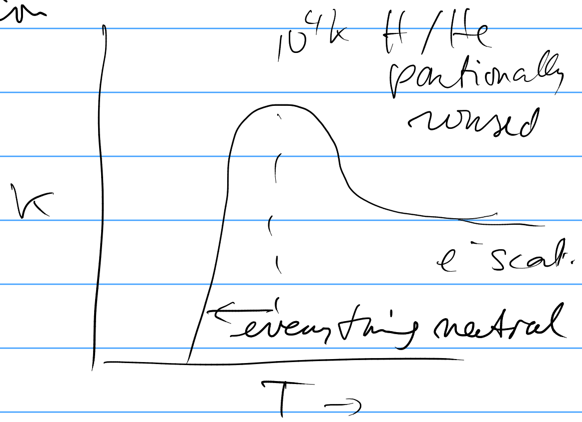


At some point,
cold enough for recombination

for SN, rec. @ $T \simeq 6000\,K$

for stars: $T \approx 10000\,K$

for CMB: $T \approx 3000\,K$



Q Effect SN?

1. photosphere further in, at higher $T = T_{rec}$
2. Extra light/energy due to recombination
3. Some radiation liberated

negative

$$L = -4\pi R_{ion}^2 v_{ion} \left(a T_{ion}^4 + \rho_{ion} Q_{ion}\right)$$

13.6 eV/nuc for H

1.3×10^{13} erg/g

$\rightarrow M X_H Q_H = 2.6 \times 10^{46}\,\text{erg}\,\frac{M}{M_\odot} X_H$

over 1 month $\sim 10^6$ $\rightarrow 10^{40}$ erg/s