Dynamical instabilities

Perturbations in P and ρ of a shell at r (m)

HE: $P_h = \int_m^M \frac{Gm}{4\pi r^4}\, dm$

continuity equation $\rho = \frac{1}{4\pi r^2}\frac{dm}{dr}$

Homologous perturbation $r' = r(1-\varepsilon)$

where $\varepsilon \ll 1$.

$$\rho' = \frac{1}{4\pi r'^2}\frac{dm}{dr'} = \frac{1}{4\pi r^2(1-\varepsilon)^2}\frac{dm}{dr}\frac{dr}{dr'}$$

$$= \frac{\rho}{(1-\varepsilon)^3} \simeq \rho(1+3\varepsilon)$$

Short timescale, assume adiabatic contraction

$$P'_{gas} = P\left(\frac{\rho'}{\rho}\right)^{\gamma_a} = P(1+3\varepsilon)^{\gamma_a} \simeq P(1+3\gamma_a\varepsilon)$$

$$P'_h = \int_m^M \frac{Gm\,dm}{4\pi r^4(1-\varepsilon)^4} \simeq P_h(1+4\varepsilon)$$

$$P'_{gas} > P'_h \Rightarrow 3\gamma_a > 4 \Rightarrow \gamma_a > \frac{4}{3}$$

($P \propto \rho^\gamma \propto R^{-3\gamma}$ HE: $P \propto R^{-4}$

$\Rightarrow \gamma \geq \frac{4}{3}$)

$$\int_0^M \left(\gamma_a - \frac{4}{3}\right)\frac{P}{\rho}\, dm < 0$$

for instability

weighed by mass, so $\gamma < \frac{4}{3}$ in core

$\Rightarrow$ star unstable

Isothermal: $\gamma = 1$ (e.g., molecular clouds,

2-8 $M_\odot$ stars with convective cores

at end of H burning ⇒ Hertzsprung gap)

Adiabatic exponent for a mixture of ionized and neutral gas and applying $\xi \sim kT$:

$$\gamma_a = \frac{20 + 49x(1-x)}{12 + 31x(1-x)}$$

$$\gamma_a < \frac{4}{3} \Rightarrow \quad 0.18 \leq x \leq 0.82$$

⇒ partially ionized gases near the ionization temperature ($\xi \sim kT$)