November 17, 2022 11:01 AM

Thermal stability

$$3\int_0^M \frac{p}{\rho}\, dm = -\Omega \qquad \text{HE}$$

Ideal gas $\quad P_{rad} \neq 0$

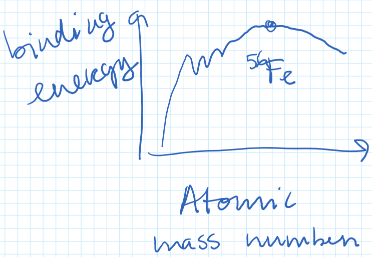
$$\frac{p}{\rho} = \frac{p_{gas}}{\rho} + \frac{P_{rad}}{\rho} = \frac{R}{\mu}T + \frac{aT^4}{3\rho} = \frac{2}{3}u_{gas} + \frac{1}{3}u_{rad}$$

with VT : $\quad U_{gas} = -\frac{1}{2}(\Omega + U_{rad})$

$$E = U_{gas} + U_{rad} + \Omega = \frac{1}{2}(\Omega + U_{rad}) = -U_{gas}$$

Also $\dot{E} = L_{nuc} - L$

Equilibrium $\quad \dot{E} = 0 \qquad L_{nuc} = L$



$L_{nuc} - L = -\dot{U}_{gas}$, $\quad L_{nuc} > L \Rightarrow \dot{U}_{gas} < 0 \Rightarrow T\downarrow$

and also $\dot{U}_{rad} < 0$

$\dot{E} = L_{nuc} - L$, $\quad L_{nuc} > L \Rightarrow \dot{E} > 0$

$\dot{E} = \dot{U} + \dot{\Omega} \Rightarrow \dot{\Omega} = \dot{E} - \dot{U}$

$\dot{\Omega} > 0 \Longrightarrow$ expansion $\Longrightarrow \bar{\rho}\downarrow$

$\Rightarrow T\downarrow$ and $\rho\downarrow$

Nuclear energy rate $\quad q = q_0 \rho^{\alpha} T^{\beta}$

$$L_{nuc} = \int_0^M q\, dm$$

Thermal instability

$L_{nuc}\uparrow \Rightarrow T\uparrow \Rightarrow P\uparrow$ (ideal gas)

$\Rightarrow$ expansion $\Rightarrow \rho\downarrow \Rightarrow T\downarrow \Rightarrow$ equilibrium

$P_{gas} \sim P_e = K\rho^{5/3}$ (or $\rho^{4/3}$ relativistic) independent of T

$$r + dr = r(1 + x\,dt)$$

$$\frac{dP}{P} = \frac{4}{3}\frac{d\rho}{\rho} \quad (*) \qquad \text{EoS} \quad P = C\rho^a T^b$$

a, b positive const.

$a = b = 1$ (ideal gas)

$a \geq 4/3$, $b \ll 1$ (degenerate)

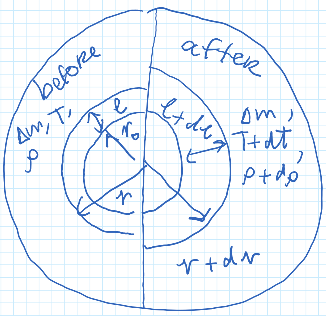
$$\Rightarrow \frac{dP}{P} = a\frac{d\rho}{\rho} + b\frac{dT}{T}$$

$$\Rightarrow \left(\frac{4}{3} - a\right)\frac{d\rho}{\rho} = b\frac{dT}{T}$$

degenerate core

$T\uparrow \Rightarrow L_{nuc}\uparrow \Rightarrow \rho\downarrow \Rightarrow T\uparrow$ unstable

Thin shell instability



Nuclear burning in thin shell

$q\uparrow$

$$\text{HE:} \quad \frac{dP}{P} = -4\frac{dr}{r}$$

$$\Delta m = 4\pi r_0^2 l\rho \Rightarrow \frac{d\rho}{\rho} = -\frac{dl}{l}, \quad dl = dr$$

$$\Rightarrow \frac{d\rho}{\rho} = -\frac{dr}{l} = -\frac{dr}{r}\frac{r}{l} \quad \text{and}$$

$$\frac{dP}{P} = 4\frac{l}{r}\frac{d\rho}{\rho} \qquad \text{with EoS}$$

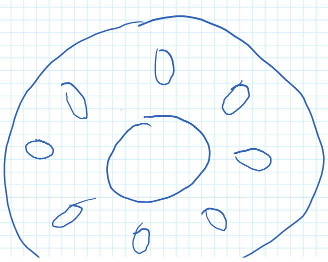
$$\left(4\frac{l}{r} - a\right)\frac{d\rho}{\rho} = b\frac{dT}{T}$$

$\uparrow$

stability $a < \frac{4l}{r}$

normal state $l \sim r \Rightarrow$ stable

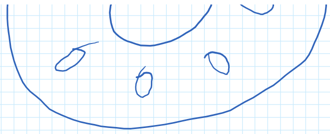
Helium flash



$\sim 0.45\, M_\odot$ $\quad T_c \sim 10^8$ K

EoS for core $P \propto \rho^{5/3}$

independent of T



independent of T

$T\uparrow \rightarrow g\uparrow \Rightarrow T\uparrow$

$L_{nuc} > 10^9 L_\odot$