## 1. Binary evolution

Most stars increase in radius as they evolve, often drastically. If in a binary, they may at some point overflow their Roche lobes, leading to mass transfer to the companion. If this is stable, mass transfer will be on the evolutionary timescale. If unstable, it can be on the dynamical or thermal timescale. Masses transfer ceases when the star stops trying to expand; in giants, this is when most of the envelope has been transferred, and the remainder becomes so tenuous that it shrinks. Thus, one generally is left with just the core of the star. This process, and variations on it, is responsible for most of the more interesting stars we observe.

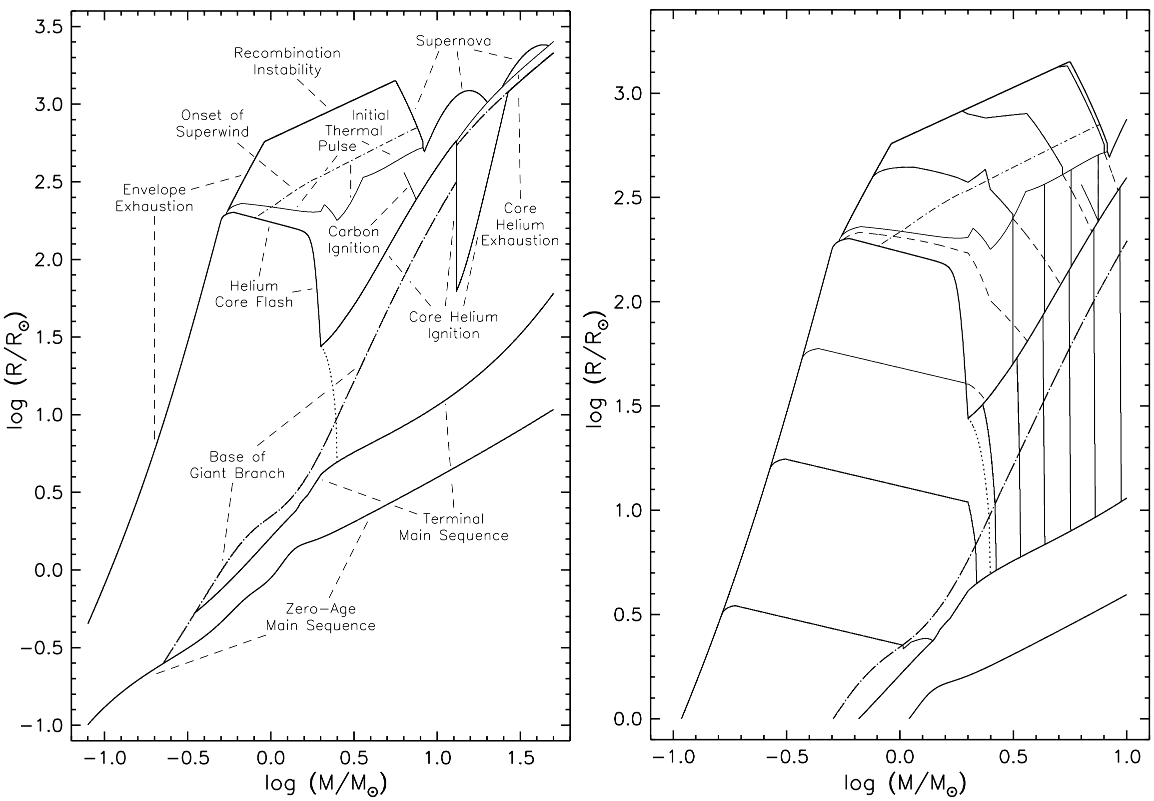


**Fig. 1.** Radius evolution of stars of various masses. In the left-hand panel, the one unmarked dotted line between 'helium core flash' and 'core helium ignition' marks the division between those helium cores (at lower masses) which evolve to degeneracy if stripped of their envelope, and those (at higher masses) which ignite helium non-degenerately and become helium stars. In the right-hand panel, core masses interior to the hydrogen-burning shell are indicated with solid lines, and dashed lines those interior to the helium-burning shell. Solid lines intersecting the base of the giant branch (dash-dotted curve) correspond to helium core masses of to 0.15, 0.25, 0.35, 0.5, 0.7, 1.0, 1.4, and 2.0 $M_\odot$; those between helium ignition and the initial thermal pulse to 0.7, 1.0, 1.4, and 2.0 $M_\odot$, and those beyond the initial thermal pulse to 0.7, 1.0, and 1.4 $M_\odot$. Dashed lines between helium ignition and initial thermal pulse correspond to carbon-oxygen core masses of 0.35, 0.5, 0.7, 1.0, and 1.4 $M_\odot$. Beyond the initial thermal pulse, helium and carbon-oxygen core masses converge, with the second dredge-up phase reducing helium core masses above $\sim 0.8\,M_\odot$ to the carbon-oxygen core. From Webbink (2008), his Figs 1 and 2.

*Mass loss and tranfer*

Consider a star that looses or transfers mass at some rate $\dot{M}$.

Effect on orbit

The angular momentum of an orbit is given by $J = (M_1 M_2/M)\sqrt{GMa}$, and thus,

$$\frac{\dot{J}}{J} = \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} - \frac{1}{2}\frac{\dot{M}}{M} + \frac{1}{2}\frac{\dot{a}}{a} \tag{1}$$

Conservative mass transfer: $\dot{M}_1 = -\dot{M}_2$, $\dot{M} = 0$, $\dot{J} = 0$. Thus,

$$\frac{\dot{a}}{a} = 2\frac{M_2 - M_1}{M_1 M_2}\dot{M}_2 = 2(q-1)\frac{\dot{M}_2}{M_2}, \tag{2}$$

where $q = M_2/M_1$ is the mass ratio between the donor (star 2) and the accretor (star 1). For donors less massive than the accretor, the orbit expands upon mass transfer (remember that $\dot{M}_2 < 0$).

Looking at the Roche lobe for a less massive donor, for which $R_L \simeq 0.46a(M_2/M)^{1/3}$ (Paczyński, 1971), one finds

$$\frac{\dot{R}_L}{R_L} = \frac{\dot{a}}{a} + \frac{1}{3}\frac{\dot{M}_2}{M_2} = 2\left(q - \frac{5}{6}\right)\frac{\dot{M}_2}{M_2}, \tag{3}$$

showing that the Roche lobe, as expected, grows a little slower than the orbital separation. (An analysis valid for all $q$ would use the approximation of Eggleton (1983), $R_L/a \simeq 0.46q^{2/3}/[0.6q^{2/3} + \ln(1+q^{1/3})]$.)

Spherically symmetric wind: $\dot{M}_2 = \dot{M}$, $\dot{M}_1 = 0$, $\dot{J} = (\dot{M}_2/M_2)(M_1/M)J$. Hence,

$$\frac{\dot{a}}{a} = 2\left(\frac{M_1\dot{M}}{M_2 M} - \frac{\dot{M}}{M_2} + \frac{\dot{M}}{2M}\right) = -\frac{\dot{M}}{M}. \tag{4}$$

Thus, for mass loss ($\dot{M} < 0$), the orbit expands.

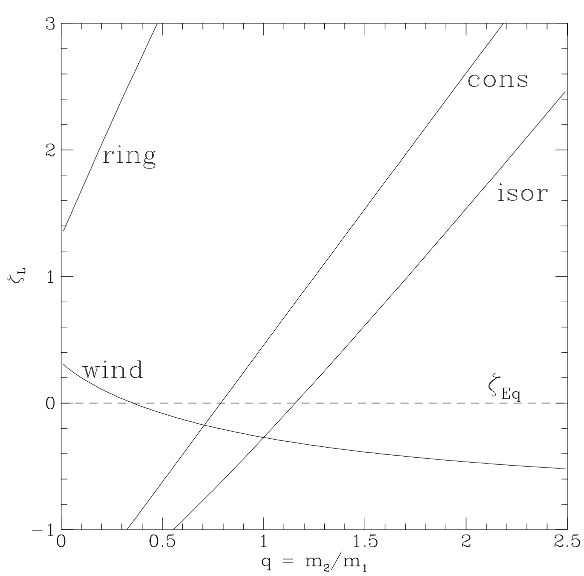


**Fig. 2.** $\zeta_L \equiv \partial \ln R_L/\partial \ln M_2$ as a function of mass ratio, with all mass transfer through a single channel: conservative (cons); isotropic wind from donor star (wind); isotropic re-emission of matter, from vicinity of 'accreting' star (iso-r). (Also shown is a ring formation, indicative of mass loss from an outer Lagrange point). From Soberman et al. (1997), their Fig. 4.

Spherically re-emitted wind: $\dot{M}_2 = \dot{M}$, $\dot{M}_1 = 0$, $\dot{J} = (\dot{M}_2/M_1)(M_2/M)J$ (idea is that accretor cannot handle mass transferred to it and re-emits it as a wind). Hence,

$$\frac{\dot{a}}{a} = 2\left(\frac{M_2\dot{M}}{M_1 M} - \frac{\dot{M}}{M_2} + \frac{\dot{M}}{2M}\right) = \frac{2q^2 - 2 - q}{1+q}\frac{\dot{M}}{M}. \tag{5}$$

Hence, orbit expands for $q < (1 + \sqrt{17})/4 = 1.28$ (with again a somewhat lower value for increasing Roche-lobe radius), i.e., it is less quickly unstable than for conservative mass transfer. For a more detailed analysis, see Soberman et al. (1997).

### Effect on stellar radius

If the mass is lost from the outside of a star, the star becomes initially smaller, but on a hydrodynamic timescale it will partially re-expand in responds to the decreased pressure. Which effect dominates depends on the internal structure of the star. Generally, for thermal envelopes, the stars shrinks inside its Roche lobe, re-expanding only on the thermal timescale, typically to nearly its original size (especially for giants). For more detail, see Hjellming & Webbink (1987). However, a complication for thermal-timescale mass transfer is that, if the secondary is substantially less massive, it cannot accrete sufficiently fast and will bloat itself. For massive stars, for $M_2/M_1 \lesssim 0.7$, this leads to contact, and almost certainly further mass loss and/or a merger (Pols, 1994; Wellstein et al., 2001; Podsiadlowski, 2010).

Completely convective stars, or stars with deep convective layers, however, increase in size upon mass loss. For completely convective stars, which are described well by polytropes with $P = K\rho^\gamma$ with $\gamma = \frac{5}{3}$ (and thus $n = 1.5$), this follows immediately from the mass radius relation: $R \propto M^{-1/3}$ (true for constant $K$, i.e., for constant entropy or completely degenerate, non-relativistic gas). Comparing this to the change in Roche lobe for conservative mass transfer, one sees that stability requires that

$$2\left(q - \frac{5}{6}\right) < -\frac{1}{3} \Leftrightarrow q < \frac{2}{3} \qquad \text{for} \quad n = 1.5. \tag{6}$$

From the work of Han & Webbink (1999), it is indeed clear that for low-mass white dwarfs, dynamical instability sets in for $q > \frac{2}{3}$. For higher mass accretors ($M_1 \gtrsim 0.3\,M_\odot$), the mass-transfer rate rapidly becomes super-Eddington, meaning some mass has to leave the system. As shown above, this implies the binary expands more and it is easier to keep mass transfer stable. Han & Webbink (1999) find that, roughly, stability requires $q \lesssim 0.7 - 0.1(M_1/M_\odot)$.

### *Common-envelope evolution*

When dynamically unstable mass transfer starts, the stars enter a common envelope. This will lead to a merger unless one envelope is relatively loosely bound, e.g., if the donor is a red giant. The process is still very uncertain, and usually an energy criterion is used to decide whether or not a complete merger occurs. We write the initial orbital energy as $E_{\rm orb,i} = GM_1M_2/2a_{\rm i}$, the final one as $E_{\rm orb,f} = GM_{1,c}M_2/a_{\rm f}$, and the envelope binding energy as $E_{\rm e} = GM_1M_{1,e}/\lambda R_{1,e}$. Taking $M_{1,e} = M_1 - M_{1,c}$, a roche-lobe filling star ($R_{1,e} = R_{\rm L}$), and assuming an efficiency $\alpha_{\rm CE} = E_{\rm e}/(E_{\rm orb,f} - E_{\rm orb,i})$, one finds a total shrinkage of the orbit,

$$\frac{a_{\rm f}}{a_{\rm i}} = \frac{M_{1,c}}{M_1}\left[1 + \frac{2}{\alpha_{\rm CE}\lambda}\frac{a_{\rm i}}{R_{\rm L}}\frac{M_1 - M_{1,c}}{M_2}\right]^{-1} \tag{7}$$

This shrinkage is usually very large. Tracing back the evolution of double helium white dwarfs, Nelemans et al. (2000), found that it cannot hold for the first mass-transfer phase. They proposed an alternative description based on angular momentum loss, but this was criticised strongly by Webbink (2008). Overall, though, the conclusion stands that for not too extreme mass ratios, mass transfer is stabilised somehow (perhaps by irradiation driven winds; Beer et al. 2007).

### *Angular momentum loss*

Two stars can be driven closer by angular-momentum loss. For gravitational radiation (in a circular orbit),

$$-\frac{\dot{J}}{J} = \frac{32G^3}{5c^5}\frac{M_1M_2(M_1+M_2)}{a^4}, \tag{8}$$

implying a merger time of $1.05\times10^7\,\mathrm{yr}(M/M_\odot)^{-2/3}(\mu/M_\odot)^{-1}(P/1\,\mathrm{hr})^{8/3}$, where $\mu = M_1M_2/(M_1+M_2)$ is the reduced mass, and $P$ the orbital period. Thus, to merge within a Hubble time requires $P \lesssim 0.5\,\mathrm{d}$.

For binaries with low-mass stars, angular momentum can also be lost by "magnetic braking" – a solar-like wind coupled to a magnetic field. This mechanism is usually described by semi-empirical relations, which are calibrated using the rotational evolution of single stars and using population synthesis models for binaries.

### *Supernova explosions*

One can solve the effect of a spherically symmetric supernova explosion by consiering that, for instantaneous mass loss, the velocities of the two stars remain the same, but their mutual attraction has decreased. Thus, the instantaneous position will become the periastron of the new orbit. For given mass loss $\Delta M$,

$$r_{\mathrm{f,peri}} = r_{\mathrm{i}} \;\Leftrightarrow\; a_{\mathrm{f}}(1-e) = a_{\mathrm{i}}, \tag{9}$$

$$v_{\mathrm{f,peri}} = v_0 \;\Leftrightarrow\; \frac{G(M_1+M_2-\Delta M)}{a_{\mathrm{f}}}\frac{1+e}{1-e} = \frac{G(M_1+M_2)}{a_{\mathrm{i}}}. \tag{10}$$

Solving this yields

$$e = \frac{\Delta M}{M_1+M_2-\Delta M}, \tag{11}$$

i.e., the orbit is unbound if $\Delta M > \frac{1}{2}(M_1+M_2)$ (as can be seen more easily from the Virial Theorem). The binary also gets a recoil kick, of

$$\Delta\gamma = \frac{M_2v_2-(M_1-\Delta M)v_1}{M_1+M_2-\Delta M} = ev_1. \tag{12}$$

Unfortunately, the assumption that supernova explosions are spherically symmetric seems rather poor, since single radio pulsars have large space velocities, of several $100\,\mathrm{km\,s^{-1}}$. As a result, binaries likely unbind even when relatively little mass is lost, and, conversely, may remain bound even if a large amount of mass is lost (indeed, the latter may be a requirement to understand low-mass X-ray binaries, in which neutron stars accrete from low-mass companions). There is fairly strong evidence, however, that some supernovae do not impart (large) kicks, possibly those due to electron capture (van den Heuvel 2010, and references therein).

### *Tidal stability*

For close binaries, tides will circularise the orbit. This is not possible if the mass ratio is too small. Stability requires that some angular momentum transfer from the orbit to the star changes the stellar rotation faster than the orbital one. Since $J_{\mathrm{orb}} = (M_1M_2/M)\sqrt{GMa} \propto \Omega^{-1/3}$ and $J_{\mathrm{star}} = I_{\mathrm{star}}\Omega \propto \Omega$, stability requires that $J_{\mathrm{orb}} > 3J_{\mathrm{star}}$. For low-mass stars, binaries with mass ratio $q \lesssim 0.09$ are unstable (Rasio, 1995).

### *Rapid rotation*

Tidal synchronisation leads to rapid rotation. For low-mass stars, this leads to increased activity, some increase in size, and a larger stellar wind (and thus angular momentum loss; magnetic breaking).

For massive stars, rotation induces mixing (de Mink et al., 2009). For fairly massive stars, just brings up nitrogen. For $M \gtrsim 50\,M_\odot$ in a $P \lesssim 2\,\mathrm{d}$ binary, centrally produced helium is efficiently mixed. As a result, these stars may burn completely to helium, and a lower-mass companion might evolve faster!

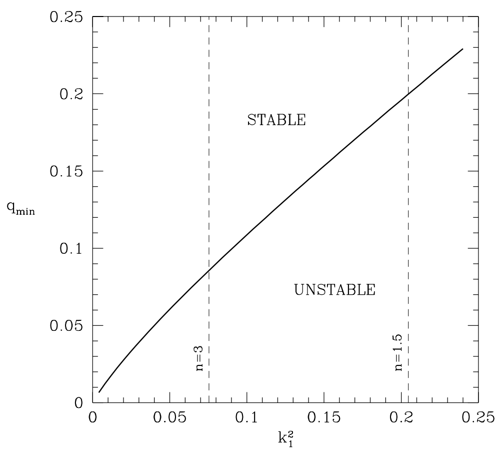


**Fig. 3.** Minimum mass ratio required for tidal stability as a function of gyration radius. From Rasio (1995), his Fig. 1.

## References

Beer, M. E., Dray, L. M., King, A. R., & Wynn, G. A. 2007, MNRAS, 375, 1000
Bhattacharya, D., & van den Heuvel, E. P. J. 1991, Phys. Rep., 203, 1
de Mink, S. E., Cantiello, M., Langer, N., Pols, O. R., Brott, I., & Yoon, S. 2009, A&A, 497, 243
Eggleton, P. P. 1983, ApJ, 268, 368
Han, Z., & Webbink, R. F. 1999, A&A, 349, L17
Hjellming, M. S., & Webbink, R. F. 1987, ApJ, 318, 794
Nelemans, G., Verbunt, F., Yungelson, L. R., & Portegies Zwart, S. F. 2000, A&A, 360, 1011

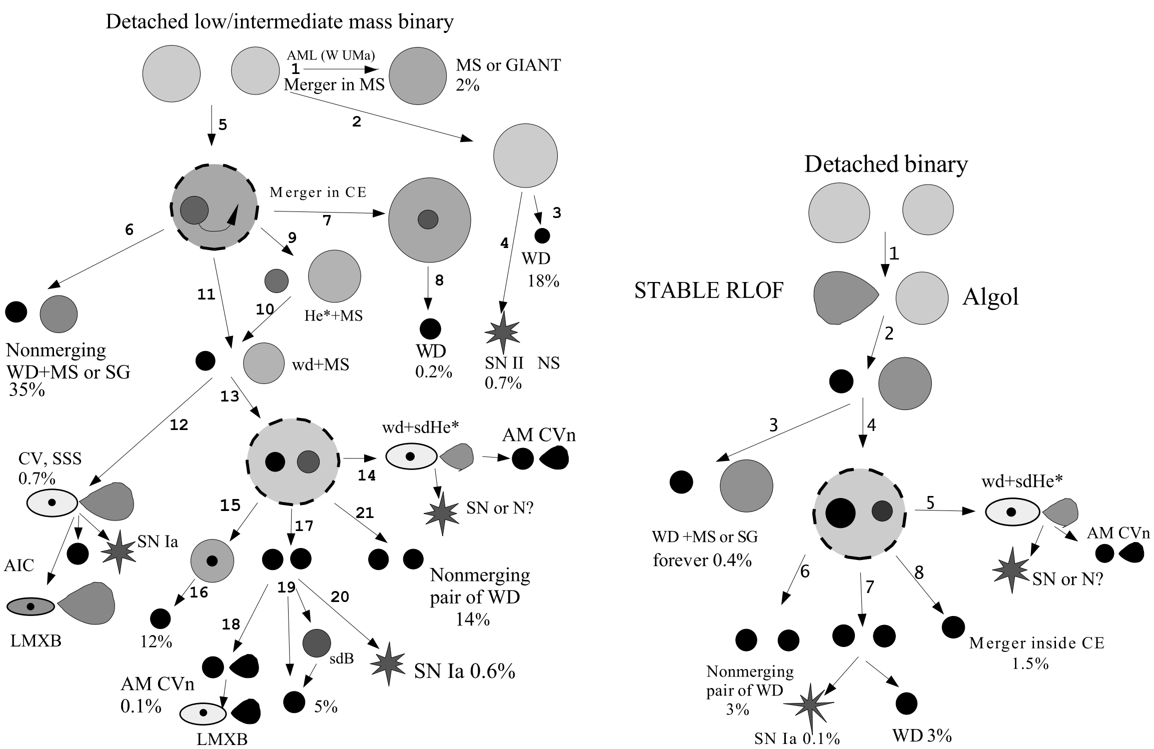


**Fig. 4.** Possible outcomes for low and intermediate-mass binaries. Yungelson (2005), Fig. 2 and 3.

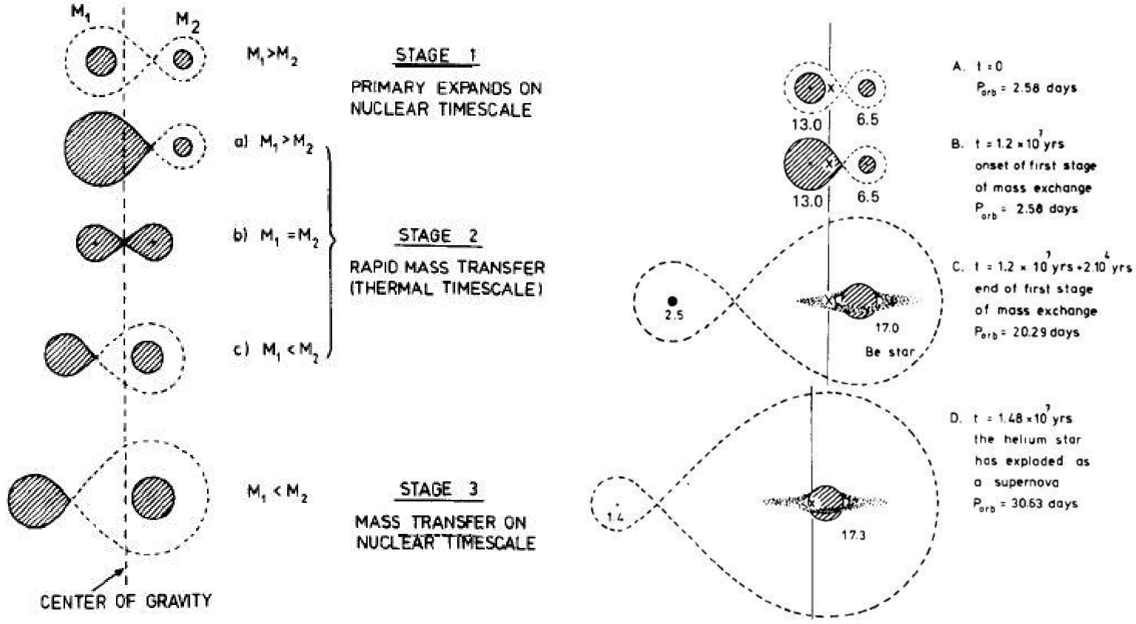


**Fig. 5.** Conservative evolution of a massive binary (left) and formation of a Be X-ray binary (right). From Bhattacharya & van den Heuvel (1991), their Figs. 24 and 25.

Paczyński, B. 1971, ARA&A, 9, 183
Podsiadlowski, P. 2010, New A Rev., 54, 39
Pols, O. R. 1994, A&A, 290, 119
Rasio, F. A. 1995, ApJ, 444, L41
Soberman, G. E., Phinney, E. S., & van den Heuvel, E. P. J. 1997, A&A, 327, 620
van den Heuvel, E. P. J. 2010, New A Rev., 54, 140
Webbink, R. F. 2008, in Astrophysics and Space Science Library, Vol. 352, Astrophysics and Space Science Library, ed. E. F. Milone, D. A. Leahy, & D. W. Hobill, 233
Wellstein, S., Langer, N., & Braun, H. 2001, A&A, 369, 939
Yungelson, L. R. 2005, in American Institute of Physics Conference Series, Vol. 797, Interacting Binaries: Accretion, Evolution, and Outcomes, ed. L. Burderi, L. A. Antonelli, F. D'Antona, T. di Salvo, G. L. Israel, L. Piersanti, A. Tornambè, & O. Straniero, 1–10

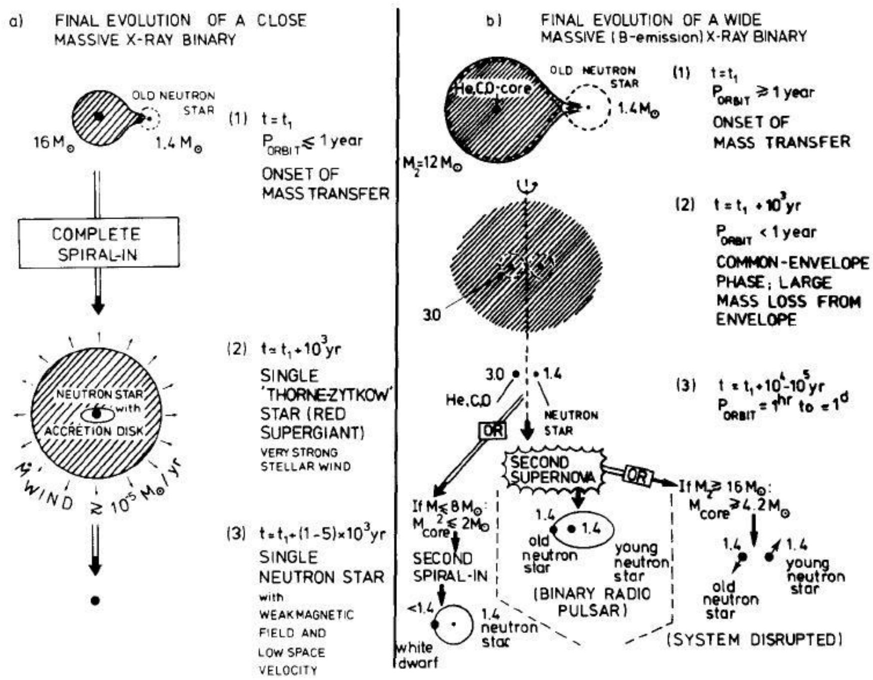


**Fig. 6.** Further evolution of X-ray binaries with short (left) and long (right) orbital periods. From Bhattacharya & Van den Heuvel (1991), their Figs. 32.